<u>AMENDMENTS TO THE CLAIMS</u>

Claims 1-18 (Canceled)

19.   (Previously Presented) A method of ~~deriving a set of closed time average turbulent moment equations for~~ <u>modelling</u> anisotropic turbulent flow in an anisotropic fluid comprising:

providing input to a general purpose computer defining<u>, for an anisotropic fluid,</u> a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

instructing the general purpose computer to calculate n^{th} order, wherein n is odd, directional kinetic energy fluxes and structure correlation equations closure relationships using $(n + 1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

<u>using the set of closed time average turbulent moment equations to predict anisotropic turbulent flow of the anisotropic fluid;</u>

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Turbulent Energy

$$\frac{\partial}{\partial t}\left[\overline{u_1'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'^2}\right]$$

$$+ \ 2\left[\overline{u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}\right]$$

$$+ \ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_1'u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_2'u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_3'u_1'^2}\right]\right]$$

$$= \ 0$$

Turbulent Shear

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]$$

$$+ \ \overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_2'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}$$

$$+ \ \overline{u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u_2}}{\partial x_3}$$

$$+ \ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_2'u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_1'u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_1'u_2'u_3'}\right]\right]$$

$$= \ 0$$

Directional Turbulent Energy Fluxes

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_1'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_1'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_1'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_1'^2}\right]$$

$$+ \ 3\left[\overline{u_1'u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'u_1'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_3'u_1'^2}\frac{\partial \overline{u_1}}{\partial x_3}\right]$$

$$- \ 3\,\overline{u_1'^2}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+ \ 3\,\overline{\overline{c_1^2}}\left[\frac{\delta}{\delta x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+ \ 3\left[\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]\right]$$

$$+ \ \frac{\partial}{\partial x_1}\left[\overline{u_1'^2u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_1'^2}\right]$$

$$+ \ \left[\overline{u_1'^2u_1'^2} - 3\,\overline{u_1'^2}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_1}$$

$$+ \ \left[\overline{u_1'u_2'u_1'^2} - 3\,\overline{u_1'u_2'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_2}$$

$$+ \ \left[\overline{u_1'u_3'u_1'^2} - 3\,\overline{u_1'u_3'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_3}$$

$$= \ 0$$

Directional Turbulent Energy Fluxes

$$\frac{\partial}{\partial t}\left[\overline{u_1' u_2'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'^2}\right]$$

$$+\; 2\left[\overline{u_1' u_2'^2}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_2' u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2' u_3'}\frac{\partial \overline{u_2}}{\partial x_3}\right]$$

$$+\; \overline{u_2' u_2'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1' u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3' u_1'^2}\frac{\partial \overline{u_1}}{\partial x_3}$$

$$-\; 2\,\overline{u_1' u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_2' u_3'}\right]\right]$$

$$-\; \overline{u_2'^2}\left[\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right]$$

$$+\; 2\left[\overline{\overline{c_2^2}}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_1 c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_2 c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'}\right]\right]$$

$$+\; \overline{\overline{c_1 c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right] + \overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right] + \overline{\overline{c_1 c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'^2}\right]$$

$$+\; 2\left[\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \overline{u_1' u_2'}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{u_2' u_3'}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'}\right]\right]$$

$$+\; \overline{u_1' u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2^2}}\right] + \overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2^2}}\right] + \overline{u_1' u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]$$

$$+\; \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2' u_2'^2}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1'^2 u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3' u_2'^2}\right]$$

$$+\; \left[\overline{u_1' u_2' u_2'^2} - 3\,\overline{u_1' u_2'}\left[\overline{u_2'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_2}$$

$$+\; \left[\overline{u_1'^2 u_2'^2} - \overline{u_1'^2}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_1' u_2'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_1}$$

$$+\; \left[\overline{u_1' u_3' u_2'^2} - \overline{u_1' u_3'}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_2' u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_3}$$

$$=\; 0$$

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'u_3'}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'u_3'}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_3'}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'}\right]$$

$$+ \ \overline{u_1'u_2'u_3'}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3'u_2'^2}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_2'u_3'^2}\frac{\partial \overline{u_1}}{\partial x_3}$$

$$+ \ \overline{u_3'u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1'u_2'u_3'}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1'u_3'^2}\frac{\partial \overline{u_2}}{\partial x_3}$$

$$+ \ \overline{u_2'u_1'^2}\frac{\partial \overline{u_3}}{\partial x_1} + \overline{u_1'u_2'^2}\frac{\partial \overline{u_2}}{\partial x_3} + \overline{u_1'u_2'u_3'}\frac{\partial \overline{u_3}}{\partial x_3}$$

$$- \ \overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_3'^2}\right]\right]$$

$$- \ \overline{u_1'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]$$

$$- \ \overline{u_2'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+ \ \overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2c_3}}\right] + \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2c_3}}\right] + \overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_2c_3}}\right]$$

$$+ \ \overline{u_1'u_2'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_3}}\right] + \overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_3}}\right] + \overline{u_2'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_3}}\right]$$

$$+ \ \overline{u_1'u_3'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_2}}\right] + \overline{u_2'u_3'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_2}}\right] + \overline{u_3'^2}d\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_2}}\right]$$

$$+ \ \overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'}\right] + \overline{\overline{c_1c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right] + \overline{\overline{c_1c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]$$

$$+ \ \overline{\overline{c_1c_3}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{\overline{c_2c_3}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \overline{\overline{c_3^2}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]$$

$$+ \ \overline{\overline{c_1c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right] + \overline{\overline{c_2^2}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'}\right] + \overline{\overline{c_2c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]$$

$$+ \ \frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'^2}\right]$$

$$+ \ \left[\overline{u_2'u_3'u_1'^2} - \overline{u_2'u_3'}\left[\overline{u_1'^2}\right] - 2\,\overline{u_1'u_2'}\left[\overline{u_1'u_3'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial \overline{\rho}}{\partial x_1}$$

$$+ \ \left[\overline{u_1'u_3'u_2'^2} - \overline{u_1'u_3'}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial \overline{\rho}}{\partial x_2}$$

$$+ \ \left[\overline{u_1'u_2'u_3'^2} - \overline{u_1'u_2'}\left[\overline{u_3'^2}\right] - 2\,\overline{u_1'u_3'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial \overline{\rho}}{\partial x_3}$$

$$= 0$$

20.   (Previously Presented) A computer readable storage medium containing a set of instructions for a general purpose computer, the set of instructions defining a method of deriving a set of closed time average turbulent moment equations for modelling anisotropic turbulent flow in an anisotropic fluid comprising:

defining, for an anisotropic fluid, a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

calculating n^{th} order , wherein n is odd, directional kinetic energy fluxes and structure correlation equations closure relationships using $(n + 1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

using the set of closed time average turbulent moment equations to predict anisotropic turbulent flow of the anisotropic fluid;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Turbulent Energy

$$\frac{\partial}{\partial t}\left[\overline{u_1'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'^2}\right]$$

$$+ 2\left[\overline{u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}\right]$$

$$+ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_1' u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_2' u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_3' u_1'^2}\right]\right]$$

$$= 0$$

Turbulent Shear

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]$$

$$+ \; \overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_2'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}$$

$$+ \; \overline{u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u_2}}{\partial x_3}$$

$$+ \; \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_2'u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_1'u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_1'u_2'u_3'}\right]\right]$$

$$= \; 0$$

Directional Turbulent Energy Fluxes

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_1'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_1'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_1'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_1'^2}\right]$$

$$+ \; 3\left[\overline{u_1'u_1'^2}\,\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'u_1'^2}\,\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_3'u_1'^2}\,\frac{\partial \overline{u_1}}{\partial x_3}\right]$$

$$- \; 3\,\overline{u_1'^2}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+ \; 3\,\overline{\overline{c_1^2}}\left[\frac{\delta}{\delta x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+ \; 3\left[\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]\right]$$

$$+ \; \frac{\partial}{\partial x_1}\left[\overline{u_1'^2 u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_1'^2}\right]$$

$$+ \; \left[\overline{u_1'^2 u_1'^2} - 3\,\overline{u_1'^2}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_1}$$

$$+ \; \left[\overline{u_1'u_2'u_1'^2} - 3\,\overline{u_1'u_2'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_2}$$

$$+ \; \left[\overline{u_1'u_3'u_1'^2} - 3\,\overline{u_1'u_3'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_3}$$

$$= \; 0$$

Directional Turbulent Energy Fluxes

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'^2}\right]$$

$$+ \ 2\left[\overline{u_1'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_2} + \overline{u_2'u_1'^2}\frac{\partial\overline{u_2}}{\partial x_1} + \overline{u_1'u_2'u_3'}\frac{\partial\overline{u_2}}{\partial x_3}\right]$$

$$+ \ \overline{u_2'u_2'^2}\frac{\partial\overline{u_1}}{\partial x_2} + \overline{u_1'u_1'^2}\frac{\partial\overline{u_1}}{\partial x_1} + \overline{u_3'u_1'^2}\frac{\partial\overline{u_1}}{\partial x_3}$$

$$- \ 2\,\overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]$$

$$- \ \overline{u_2'^2}\left[\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+ \ 2\left[\overline{\overline{c_2^2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{\overline{c_1c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{\overline{c_2c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]$$

$$+ \ \overline{\overline{c_1c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right] + \overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right] + \overline{\overline{c_1c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'^2}\right]$$

$$+ \ 2\left[\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \overline{u_1'u_2'}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{u_2'u_3'}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]$$

$$+ \ \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2^2}}\right] + \overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2^2}}\right] + \overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]$$

$$+ \ \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_2'^2}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1'^2u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_2'^2}\right]$$

$$+ \ \left[\overline{u_1'u_2'u_2'^2} - 3\,\overline{u_1'u_2'}\left[\overline{u_2'^2}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial\overline{\rho}}{\partial x_2}$$

$$+ \ \left[\overline{u_1'^2u_2'^2} - \overline{u_1'^2}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1'u_2'}\left[\overline{u_1'u_2'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial\overline{\rho}}{\partial x_1}$$

$$+ \ \left[\overline{u_1'u_3'u_2'^2} - \overline{u_1'u_3'}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial\overline{\rho}}{\partial x_3}$$

$$= 0$$

$$\frac{\partial}{\partial t}\left[\overline{u_1' u_2' u_3'}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2' u_3'}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2' u_3'}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2' u_3'}\right]$$

$$+ \overline{u_1' u_2' u_3'}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3' u_2'^2}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_2' u_3'^2}\frac{\partial \overline{u_1}}{\partial x_3}$$

$$+ \overline{u_3' u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2' u_3'}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1' u_3'^2}\frac{\partial \overline{u_2}}{\partial x_3}$$

$$+ \overline{u_2' u_1'^2}\frac{\partial \overline{u_3}}{\partial x_1} + \overline{u_1' u_2'^2}\frac{\partial \overline{u_2}}{\partial x_3} + \overline{u_1' u_2' u_3'}\frac{\partial \overline{u_3}}{\partial x_3}$$

$$- \overline{u_1' u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1' u_3'}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_2' u_3'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_3'^2}\right]\right]$$

$$- \overline{u_1' u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_2' u_3'}\right]\right]$$

$$- \overline{u_2' u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right]$$

$$+ \overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2 c_3}}\right] + \overline{u_1' u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2 c_3}}\right] + \overline{u_1' u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_2 c_3}}\right]$$

$$+ \overline{u_1' u_2'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1 c_3}}\right] + \overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1 c_3}}\right] + \overline{u_2' u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1 c_3}}\right]$$

$$+ \overline{u_1' u_3'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1 c_2}}\right] + \overline{u_2' u_3'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1 c_2}}\right] + \overline{u_3'^2}d\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1 c_2}}\right]$$

$$+ \overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2' u_3'}\right] + \overline{\overline{c_1 c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2' u_3'}\right] + \overline{\overline{c_1 c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2' u_3'}\right]$$

$$+ \overline{\overline{c_1 c_3}}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_2 c_3}}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_3^2}}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'}\right]$$

$$+ \overline{\overline{c_1 c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_3'}\right] + \overline{\overline{c_2^2}}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_3'}\right] + \overline{\overline{c_2 c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]$$

$$+ \frac{\partial}{\partial x_1}\left[\overline{u_2' u_3' u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_3' u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_2' u_3'^2}\right]$$

$$+ \left[\overline{u_2' u_3' u_1'^2} - \overline{u_2' u_3'}\left[\overline{u_1'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_1' u_3'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial \overline{\rho}}{\partial x_1}$$

$$+ \left[\overline{u_1' u_3' u_2'^2} - \overline{u_1' u_3'}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_2' u_3'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial \overline{\rho}}{\partial x_2}$$

$$+ \left[\overline{u_1' u_2' u_3'^2} - \overline{u_1' u_2'}\left[\overline{u_3'^2}\right] - 2\,\overline{u_1' u_3'}\left[\overline{u_2' u_3'}\right]\right]\frac{1}{\overline{\overline{\rho}}}\frac{\partial \overline{\rho}}{\partial x_3}$$

$$= 0$$